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HEAT TRANSFER BY FORCED CONVECTION FROM A CYLINDER TO WATER IN CROSSFLOW

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Abstract—This paper presents the results of an experimental investigation of heat transfer by forced convection from a cylinder to water in crossflow in the range of Reynolds numbers from approximately 10^4 to 10^5. The diameter of the test cylinder was 0·4375 in, and the temperature difference varied from 4 to 10 degF. The experimental data are in close agreement (mean deviation = 1 per cent) with the McAdams correlation, namely,

$$(Nu)_f = [0{\cdot}35 + 0{\cdot}56\,(Re)_f^{0{\cdot}52}]\,(Pr)_f^{0{\cdot}30}.$$

McAdams derived his correlation equation from experimental data taken in the limited range of crossflow Reynolds numbers from 10^{-1} to 2×10^2. Purves and Brodkey infer from the Colburn analogy that McAdams' correlation is valid in the range of Reynolds number from 10^2 to 10^4. Therefore, on the basis of the present experimental data, it is concluded that McAdams' equation is valid in the extended range of Reynolds numbers from 10^{-1} to 10^5.

The present experimental data are equally well represented by

$$(Nu)_f = [0{\cdot}35 + 0{\cdot}34\,(Re)_f^{0{\cdot}50} + 0{\cdot}15\,(Re)_f^{0{\cdot}58}]\,(Pr)_f^{0{\cdot}30}.$$

In the latter equation, the term in $(Re)_f^{0{\cdot}50}$ represents the heat transfer through the laminar boundary layer on the front portion of the cylinder, and the term in $(Re)_f^{0{\cdot}58}$ accounts for the contribution to the total heat transfer from the back portion of the cylinder, where separation occurs. Either of the two equations given here may be used for design calculations, but the second equation is preferable in the the sense that it is more logically related to the physical processes involved.

NOMENCLATURE

- a, b, functions of Pr [see equation (5)];
- C, C' constants;
- D, diameter of water tunnel test section;
- d, diameter of heat-transfer cylinder;
- g, indicates functional relationship;
- h, heat-transfer coefficient;
- l, effective length of heat-transfer cylinder (see Fig. 4);
- m, n, constants;
- Nu, Nusselt number;
- Pr, Prandtl number;
- q, mean convective heat flux;
- Re, Reynolds number;
- t, temperature;
- V, velocity (corrected for blockage);
- x, constant.

Greek symbols

- α, β, constants;
- Δt, difference in temperature between test cylinder and bulk of water: $\Delta t = (t_s - t_a)$;
- μ, viscosity.

Subscripts

- a, refers to conditions at ambient temperature (bulk water temperature), t_a;
- s, refers to conditions at test cylinder surface temperature, t_s;
- f, refers to conditions at arithmetic mean film temperature, $t_f = (t_s + t_a)/2$.

INTRODUCTION

THE PROBLEM of heat transfer by forced convection from single cylinders to *gases* has been studied in depth and extensive data have been published on this subject; but for the analogous case of heat transfer to *water* the available information is relatively meager. In fact, until the present decade, no data for heat transfer from cylinders to water (or other liquids) for crossflow Reynolds numbers exceeding 300 had appeared in the open literature. This lack of information at higher Reynolds numbers—more specifically, in the range from 10^4 to 10^5—was a serious scientific and engineering design deficiency, and for this reason several investigators have recently studied this problem. Unfortunately, there are discrepancies between the published results of these investigations. The present study may help resolve these discrepancies.

SURVEY OF THE LITERATURE

The earliest comprehensive data for forced convection heat transfer from cylinders to liquids in crossflow are those of Davis [1], who employed several sizes of electrically heated wires and four hydrocarbon oils having a wide range of viscosities; the range of the Reynolds number in Davis' work was from 0·1 to 200. Ulsomer [2] correlated Davis' data for liquids and those of several investigators on air by an equation of the form

$$(Nu)_f = C\,(Pr)_f^m (Re)_f^n, \qquad (1)$$

where C, m, and n are numerical constants having the following values: $m = 0{\cdot}31$; $C = 0{\cdot}91$ and $n = 0{\cdot}385$ for $0{\cdot}1 < (Re)_f < 50$; $C = 0{\cdot}6$ and $n = 0{\cdot}5$ for $50 < (Re)_f < 10^4$. Kramers [3] analysed the results of Davis for liquids and of many others with air and correlated these data by an equation similar in a form to equation (1), but containing an additional term as follows:

$$(Nu)_f = C'(Pr)_f^{m'} + C(Pr)_f^m\,(Re)_f^n, \qquad (2)$$

where the values of the numerical constants C', m', C, m and n are 0·42, 0·2, 0·57, 0·33 and 0·5 respectively. Subsequently, Piret *et al.* [4] obtained data for water for Reynolds numbers ranging from 0·8 to 8 and correlated their data by equation (1) with $C = 0{\cdot}965$, $m = 0{\cdot}30$ and $n = 0{\cdot}28$. McAdams [5] compiled the data of Davis and Piret and concluded that all these data can be satisfactorily correlated by equation (2) with $C' = 0{\cdot}35$, $C = 0{\cdot}56$, $m' = m = 0{\cdot}30$, and $n = 0{\cdot}52$. Since McAdams' correlation has special significance in the present work it will be stated here explicitly:

$$(Nu)_f = [0{\cdot}35 + 0{\cdot}56\,(Re)_f^{0{\cdot}52}]\,(Pr)_f^{0{\cdot}3}. \qquad (3)$$

In 1961 Purves and Brodkey [6] conducted an experimental study of forced convection heat transfer to water flowing normal to a cylinder at Reynolds numbers in the neighborhood of 10^4. They were unable to obtain consistent data, due to certain experimental difficulties which they encountered in the construction of suitable test specimens; however, in order to check the order of magnitude of their experimental results, they calculated the heat-transfer coefficient from friction factor data using the Colburn analogy. They found that the Colburn analogy gave results which coincided exactly with McAdams' correlation at a Reynolds number of about 100, and they concluded that McAdams' correlation can be extrapolated to Reynolds numbers of about 10^4.

An extensive study of the extant data on heat transfer from cylinders to crossflow in air has been made by Douglas and Churchill [7], who proposed a correlation in the following form:

$$(Nu)_f = a\,(Re)_f^{0{\cdot}50} + b\,(Re)_f. \qquad (4)$$

The reasoning behind equation (4) is that the first term on the right represents the heat transfer through the laminar boundary layer on the front portion of the cylinder and the second term represents heat transfer from the rear portion, where separation occurs. Van der Hegge Zijnen [8] has made a similar proposal. The quantities a and b in equation (4) are not necessarily constants; they are generally taken proportional to $(Pr)^m$, where m is a constant between 0·3 and 0·4. More recently, Richardson [9] has suggested that the heat transfer to the separated region is a function of $(Re)_f^{0{\cdot}67}$, and with this refinement, equation (4) becomes:

$$(Nu)_f = a\,(Re)_f^{0{\cdot}50} + b\,(Re)_f^{0{\cdot}67}. \qquad (5)$$

Perkins and Leppert [10] have conducted an experimental investigation of forced convection heat transfer from a uniformly heated cylinder to water and ethylene glycol in crossflow for Reynolds numbers from 40 to 10^5 and Prandtl numbers from 1 to 300. They showed that their data could be correlated adequately by slight modifications of either equations (4) or (5), and with numerical constants as in the following:

$$(Nu)_a \left(\frac{\mu_s}{\mu_a}\right)^{0\cdot25} = [0\cdot53\,(Re)_a^{0\cdot50} + 0\cdot0019\,(Re)_a]\,(Pr)_a^{0\cdot40}, \quad (6)$$

$$(Nu)_a \left(\frac{\mu_s}{\mu_a}\right)^{0\cdot25} = [0\cdot30\,(Re)_a^{0\cdot50} + 0\cdot10\,(Re)_a^{0\cdot67}]\,(Pr)_a^{0\cdot40}. \quad (7)$$

The authors state that equation (6) may give erroneously large values of the Nusselt number for Reynolds numbers greater than 10^5, and they prefer to represent their data by equation (7). The numerical coefficients in equations (6) and (7) were determined by using a "mean area" correction for blockage [see the section on Calculations and Results (p. 1001) for definition]; in a subsequent related publication, Perkins and Leppert [11] redetermined these coefficients on the basis of an experimental correction for blockage. This experimental blockage correction agrees with the well-known "solid blocking" and "wake blocking" corrections. The redetermined coefficients are indicated in equations (6′) and (7′).

$$(Nu)_a \left(\frac{\mu_s}{\mu_a}\right)^{0\cdot25} = [0\cdot57\,(Re)_a^{0\cdot50} + 0\cdot0022\,(Re)_a]\,(Pr)_a^{0\cdot40}, \quad (6')$$

$$(Nu)_a \left(\frac{\mu_s}{\mu_a}\right)^{0\cdot25} = [0\cdot31\,(Re)_a^{0\cdot50} + 0\cdot11\,(Re)_a^{0\cdot67}]\,(Pr)_a^{0\cdot40}. \quad (7')$$

Nusselt numbers calculated from equation (7′) are about 65 per cent higher than comparable values calculated from McAdams' correlation.

Perkins and Leppert [11] have obtained experimental data on local heat transfer from several uniformly heated cylinders to water in crossflow. An analysis of local heat transfer to the laminar boundary layer on the forward portion of a cylinder is also presented in [11], and the results of the analysis are compared with the experimental data and with the results of previously published analyses. The experimental heat-transfer data in the laminar region, after being corrected for blockage, are about 20 per cent higher than the analytical prediction. The prediction, however, is in good agreement with previously published analytical results. One very interesting aspect of the comparison of analyses made by Perkins and Leppert is the fact that the theoretically derived local heat-transfer coefficients in the region of laminar flow for a cylinder in crossflow are shown to be very nearly the same (less than 5 per cent different), whether the cylinder is isothermal or uniformly heated.

The disparity between the analytical predictions and experimental results in [11] are attributed by the authors to the influence of free stream turbulence. Measurements taken with a hot film anemometer indicate an average turbulence level of $2\cdot9 \pm 0\cdot5$ per cent; however, the authors conclude that both their average and local heat-transfer results appear to be more in line with a 1 per cent turbulence level. Perkins and Leppert [10] estimated the turbulence intensity at the location of their test cylinders to be about 1·1 per cent on the basis of the linear decay law of Batchelor and Townsend; they hypothesize in [11] that the disparity between the measured 2·9 per cent intensity of turbulence and the "predicted" level of 1·1 per cent may be due to the effect of the walls in the relatively narrow channel which they employed.

There has been considerable divergence of opinion as to how best to account for variations in fluid properties—particularly the viscosity—in convective heat transfer; for example, some authors recommend that the viscosity, μ, in the Reynolds number be evaluated at the ambient temperature, and then add a separate correction factor to the correlation in the form $(\mu_s/\mu_a)^x$, where (μ_s/μ_a) is the ratio of the viscosity at the temperature of the heat-transfer surface to the viscosity at the ambient temperature, and x is a constant [see equations (6) and (7)]; others recommend that fluid properties be evaluated at the integrated mean film temperature; still others favor the approach indicated in equations (1), (2) and (3), where the fluid properties are evaluated at the arithmetic mean film temperature, $t_f = (t_s + t_a)/2$. Douglas and Churchill [7] have made a careful and extensive study of this question and report that convective heat transfer from cylinders is best correlated by evaluating the fluid properties at the arithmetic mean film temperature.

Douglas and Churchill also make an important point concerning the general theory of dynamic similarity, upon which all heat transfer correlation equations depend. The point is expressed in their paper as follows:

> "Jacob has pointed out that for complete dynamic similarity between two nonisothermal systems the same ratios must exist between the significant physical properties at geometrically equivalent points. A lack of similarity will necessarily exist for any two fluids whose properties do not vary identically with temperature. A liquid and a gas fail in this respect, but two gases provide reasonable similarity."

Thus, in theory, the same correlation equation cannot be expected to predict Nusselt numbers for both gases and liquids, as is frequently assumed. This is not to say that the *general form* of the correlation equations is necessarily different for liquids and gases, but it does imply that the constants—such as C', m', C, m and n in equation (2)—will be different for liquids and gases. The reason why the same correlation equations have been used to predict Nusselt numbers for both gases and liquids is probably because other experimental factors—such as free stream turbulence—have masked the errors incurred by this procedure. For most practical design purposes the lack of exact dynamic similarity between nonisothermal systems of gases and liquids may not be important, but for understanding the physical processes involved in convective heat transfer, this factor must be taken into consideration.

APPARATUS AND PROCEDURE

Water tunnel

The experiments in this study were performed

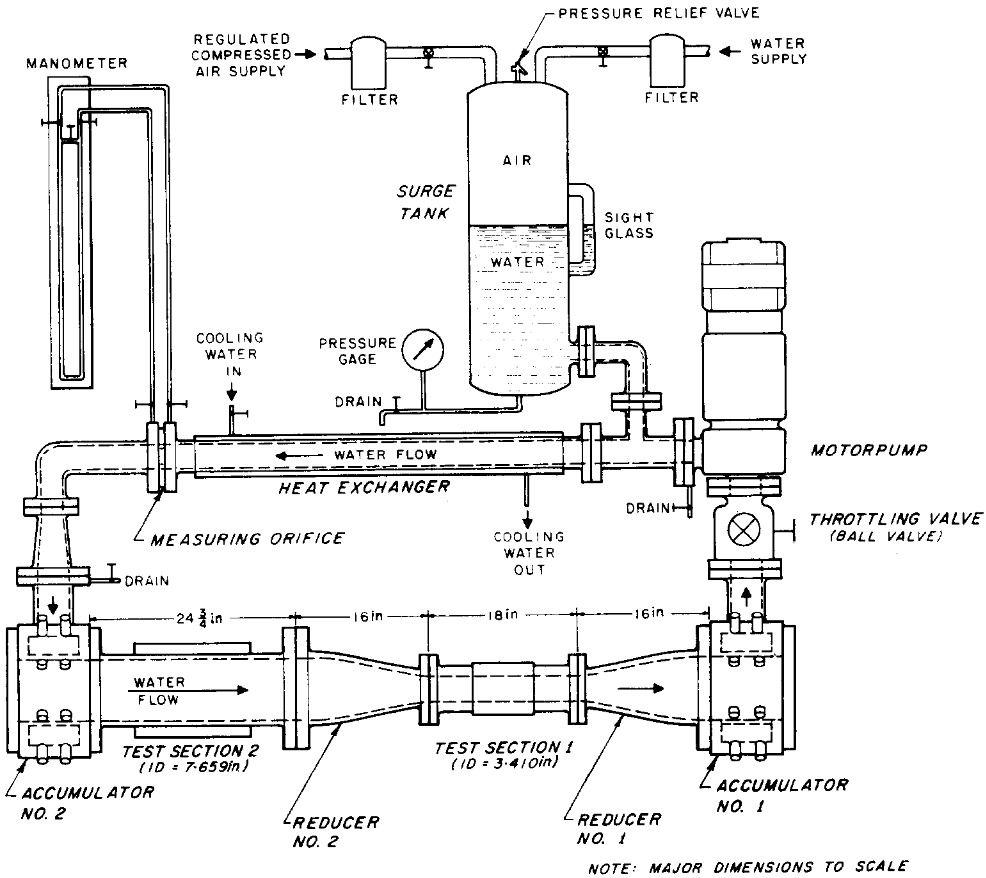


FIG. 1. Water tunnel.

in the water tunnel, shown schematically in Fig. 1. The tunnel consists of a closed loop through which water can be pumped at controlled and measured velocities; all components in contact with water are made of either stainless steel, copper, bronze, Teflon or rubber. The tunnel contains two circular test sections, a surge tank for reducing pump-induced pulsations, a heat exchanger for controlling the bulk temperature of the water in the tunnel, and a measuring orifice for determining the velocity of flow. In order to obtain maximum experimental flow velocities (maximum Reynolds numbers), the heat-transfer data were taken in the smaller of the two test sections (Test Section 1; I.D. = 3·410 in). In this arrangement, Test Section 2 and Reducer 2 act as calming sections for the flow before it enters Test Section 1.

Prior to taking heat-transfer data, the velocity at the center of Test Section 1 was measured with a small pitot tube, and these measurements were related to the pressure drop (manometer readings) across the measuring orifice; by this procedure, the velocity at the center of Test Section 1 could be subsequently inferred with an accuracy of about 1 per cent from readings of the manometer associated with the measuring orifice. The pitot tube was also used to determine velocity profiles in Test Section 1. It was found that all velocity profiles were flat in a region extending radially outward $\frac{1}{2}$ in from the center of Test Section 1; the significance of this finding will become apparent shortly. The maximum velocity in Test Section 1, when unobstructed by the heat-transfer test specimen, was 13·4 ft/s.

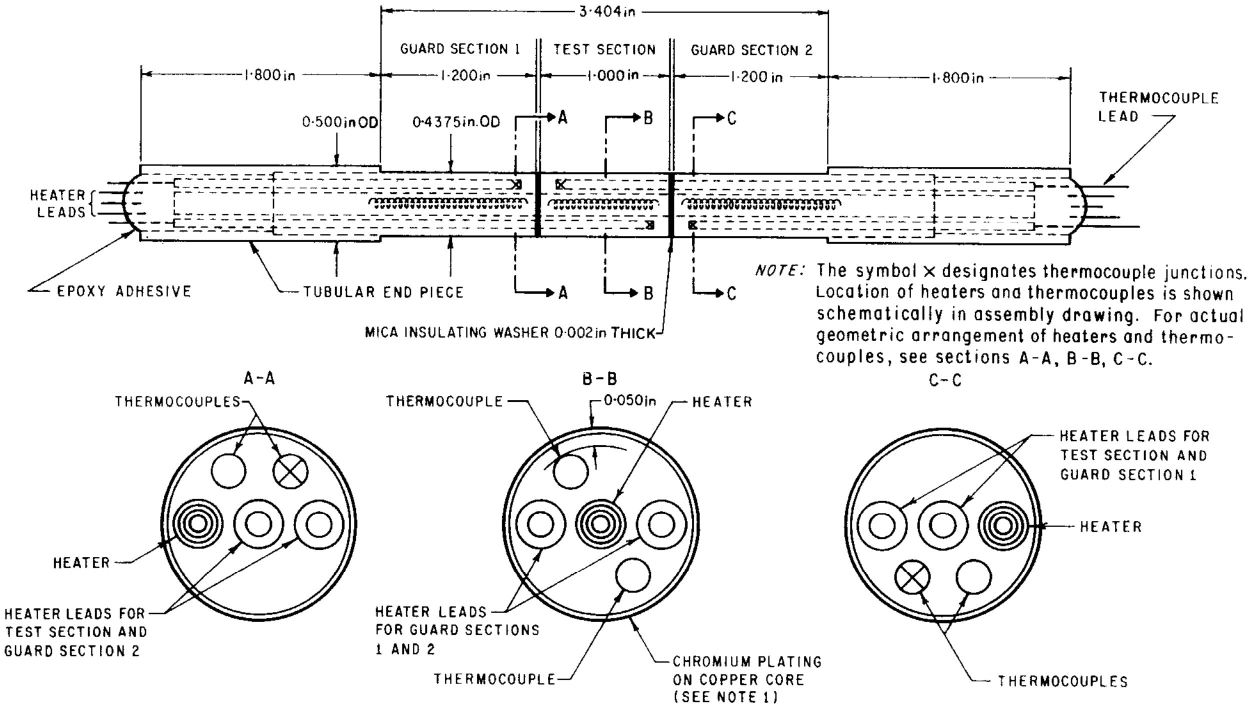
The components labeled Accumulators in Fig. 1 are toroidal sections which convey the flowing water into and out of the test sections of the tunnel, as shown by the arrows in Fig. 1; this conveyance is achieved through a number of symmetrically placed radial tubes (shown schematically in perspective affixed to the Accumulators in Fig. 1), such that the flow enters Test Section 2 and leaves Reducer 1 without introducing the asymmetry in the flow which ordinary 90° elbows would cause. A thermocouple was inserted into Accumulator 1 through one of the radial tubes for the purpose of measuring the bulk water temperature.

Heat-transfer cylinder

The heat-transfer cylinder used in these experiments was constructed in accordance with the "guarded test section" technique. In this technique, the test specimen consists of three cylindrical sections—a "test section" flanked by two "guard sections"—which are thermally insulated from one another, as shown in Fig. 2; each section contains a separate electric resistance heater and thermocouples to measure its temperature. The guard sections contain one thermocouple each, and the test section contains two thermocouples; the readings of the two thermocouples in the test section were found to be equal in all tests. The thermocouples and heaters are mounted in hypodermic tubes which are inserted longitudinally into the test and guard sections through accurately drilled holes; these hypodermic tubes serve to align the three heated sections and stiffen the composite structure. Each of the three hypodermic tubes which contains a heater threads through all three sections of the test cylinder and is embedded at its extremities in masses of epoxy adhesive which fill the tubular end pieces of the test cylinder (see Fig. 2); thus, these three heater-carrying hypodermic tubes serve the additional function of holding together the several parts of the test cylinder.

The function of the guard sections is to prevent the transfer of heat by conduction from the centrally located test section to its supporting structure; this is achieved operationally by adjusting the electrical input to the guard heaters until the temperatures of the guard sections are equal to the temperature of the central test section. Thus, when the temperatures of all three heated sections are equal, the electrical input to the central test section is equal to the heat transfer by convection from its surface. Electrical power was delivered to the three heaters by individual Variacs, and the power dissipated in the central test section was measured with an accuracy of about 1 per cent. A portable precision potentiometer was used in conjunction with the thermocouples, which provided temperature readings having an accuracy of at least 0·05 degF.

In order to achieve a uniform surface temperature, the cores of the heated sections of the test



NOTES: 1 Test cylinder sections made by plating copper rod with chromium (OD of copper core = 0·410 in OD of cylinder section = 0·4375 in)

2 Thermocouples installed inside of 14-gauge, thin-walled hypodermic tubing (0·083 in OD; 0·067 in ID)

3 Heaters installed inside of 12-gauge, thin-walled hypodermic tubing (0·109 in OD; 0·091 in ID)

FIG. 2. Heat-transfer test cylinder.

cylinder were made of copper, because this metal has high thermal conductivity. However, in order to provide a hard, non-oxidizing surface, a thin layer of chromium was plated onto these copper cores. The plated pieces were centerless ground to a final diameter of 0·4375 in. The test cylinder was inserted across Test Section 1 of the water tunnel with the aid of two special plugs which contained O-rings for sealing and which were screwed into threaded holes located on opposite sides of Test Section 1 (not shown in Fig. 1); the physical arrangement was such as to leave the entire heated length of the test cylinder exposed to the convective flow. The heated length of test cylinder (3·404 in—see Fig. 2) is practically identical to the internal diameter of Test Section 1 (3·410 in—see Fig. 1).

The guarded test section technique is a very accurate method for measuring convective heat-transfer coefficients; first, because it practically eliminates spurious heat flows by conduction from the data-producing test section and along thermocouple lead wires; and second, because the heated test section is suspended in the central portion of the duct wherein convection occurs and is, therefore, unaffected by the velocity gradients which inevitably occur near the duct walls. In the present case the central test section was 1 in long and, as has been mentioned above, the velocity profiles were found to be flat over this length, within the sensitivity of the instrumentation.

The one major limitation in the design of the test cylinder shown in Fig. 2 is the limited power handling capability of the electrical coils installed in the three heated sections. The geometry of the design demands that these coils be not much greater than $\frac{1}{16}$ in, and to fabricate, support, accurately locate and insulate a high-performance coil of this size inside of a small hypodermic tube is a delicate matter (see Fig. 3 for details). For the design shown in Figs. 2 and 3, the coil inside the 1 in long central test section was capable of dissipating not more than 60 W of electrical power without burning out. As a safety measure, no more than 50 W were delivered to the central test section during heat-transfer tests; this resulted in differences in temperature between the thermocouples in the test cylinder and the bulk of water ranging from 4 to 10 degF. Since the temperature difference was determined from two readings, each of which was measured with an accuracy of at least 0·05 degF, the combined maximum error in the measurements of temperature differences is 0·1 degF (2·5 per cent).

CALCULATIONS AND RESULTS

Velocity blockage correction

The calibration technique described in the foregoing section provided a means for determing the velocity at the center of Test Section 1 with an accuracy of about 1 per cent *in the absence of the heat-transfer cylinder*. When the test cylinder is inserted into the water tunnel, it obstructs the flow and necessitates a correction of the indicated velocity. Perkins and Leppert [10] describe three different ways in which this correction can be made; the method of Vliet and Leppert [12] has been adopted here.

In the method of Vliet and Leppert a mean area for blocked flow, A_m, is defined and the corrected velocity, V, is obtained from the unobstructed velocity by using the continuity equation as follows:

$$V = \frac{A_{\text{unobstructed}}}{A_m} (V_{\text{unobstructed}}). \tag{8}$$

The mean area for blocked flow, A_m, is defined as that area, which, when multiplied by the test specimen diameter, is equal to the net volume of fluid at the location of the test specimen. Thus, referring to Fig. 4,

$$A_m d = \left(\frac{\pi D^2}{4}\right) d - \left(\frac{\pi d^2}{4}\right) l. \tag{9}$$

When D is large in comparison to d, as is the case here, l may be taken equal to D. Since $A_{\text{unobstructed}} = (\pi D^2/4)$, equation (8) may be written as follows:

$$V = \frac{D^2}{D^2 - dl} (V_{\text{unobstructed}}).$$

By inserting the numerical values $D = l = 3{\cdot}410$ in and $d = 0{\cdot}4375$ in into the foregoing expression, one obtains $V = 1{\cdot}146\,(V_{\text{unobstructed}})$.

Temperature correction

In order to determine the surface temperature

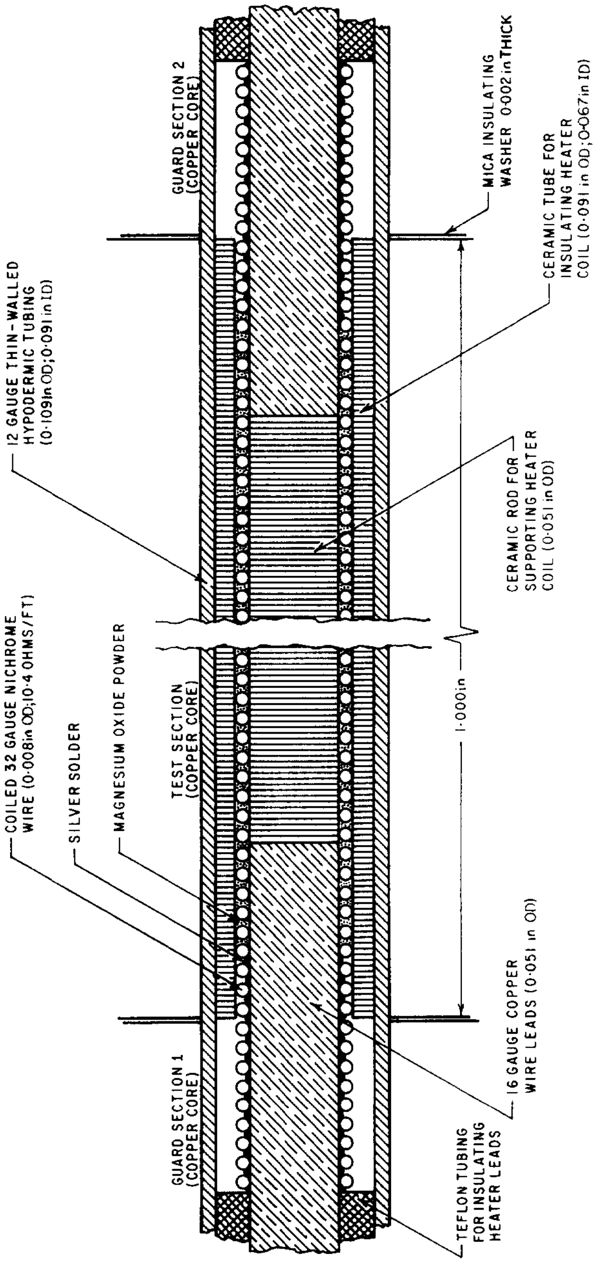


FIG. 3. Detail of test section heater.

of the test cylinder, the readings of the thermocouples embedded in the test cylinder were corrected for the small radial temperature drop across the layers of copper and chromium which lie between them and the surface of the cylinder (see Fig. 3, Section BB). This radial temperature drop, Δt_r, was calculated by assuming that the simple conduction equation for radial heat transfer through composite cylinders [13] is applicable. For conditions of the present experiment, the value of Δt_r varied from 0·54 to 0·83 degF; this represents from 5·0 to 12 per cent of the measured temperature difference between the thermocouples in the test cylinder and the ambient fluid. The method used here to calculate Δt_r is considered to be an approximation, because of the simplifying assumptions upon which it is based.

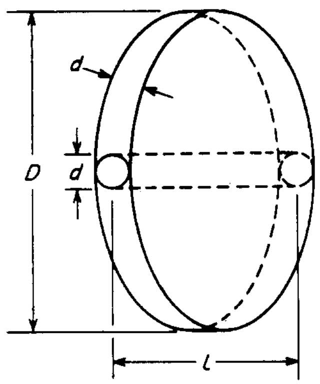


FIG. 4. Geometry for calculating mean area for blocked flow.

Heat-transfer correlations

Six heat-transfer tests were performed at different Reynolds numbers ranging from 11 420 to 63 200. A complete tabulation of all measured quantities and Reynolds numbers are presented in Table 1.

Table 2 contains comparisons of the Nusselt number calculated from equations (3) and (7′) with corresponding experimentally determined values. The values of $(Nu)_f$ calculated from equation (3), McAdams' correlation, are in excellent agreement with the experimental results; the deviation for any single run does not exceed 3·1 per cent, and the mean deviation for all six runs is 1·3 per cent. The experimental points and the McAdams correlation are plotted in Fig. 5. The deviations of $(Nu)_a$ calculated from Perkins and Leppert's correlation, equation (7′),

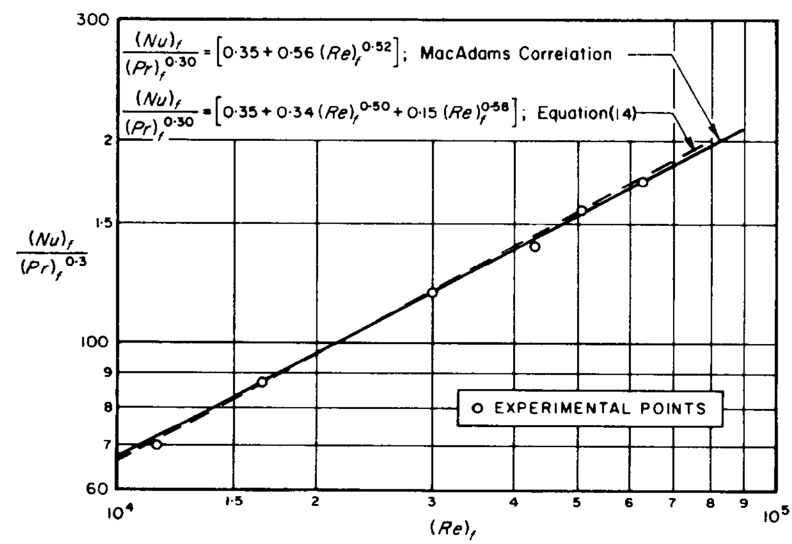


FIG. 5. Heat transfer by forced convection from a cylinder to water in crossflow.

Table 1. Experimental data

Run No.	q $\left(\frac{\text{Btu}}{\text{h ft}}\right)$	t_s (degF)	t_a (degF)	Δt (degF)	t_f (degF)	V $\left(\frac{\text{ft}}{\text{s}}\right)$	$(Re)_f$	$(Re)_a$	h $\frac{\text{Btu}}{h\ \text{ft}^2\ \text{degF}}$
1	17875	87·07	80·00	7·07	83·53	12·08	50300	48200	2525
2	10725	87·20	79·72	7·48	83·46	3·99	16630	15860	1437
3	10725	89·96	80·42	9·52	85·19	2·68	11420	10780	1126
4	10725	84·74	80·02	4·72	82·38	10·42	43000	41600	2275
5	10725	85·67	80·08	5·59	82·87	7·29	30200	29200	1921
6	10725	84·20	80·45	3·75	82·32	15·34	63800	61700	2865

from the corresponding experimental values are from +55·3 to +76·3 per cent, with a mean deviation of +66·2 per cent; thus, the values calculated from equation (7′) are consistently and significantly higher than the present experimental values. Possible reasons for this discrepancy are given in the section on Discussion of Results.

The suggestion put forth by Douglas and Churchill, and more recently by Richardson, that the contributions to the total heat transfer from the front and back portions of a cylinder in crossflow should be accounted for separately in the correlation equation is supported both by theory and experiment. This suggestion is adopted here with certain modifications, for reasons explained in the following.

If the idea of representing the experimental heat-transfer measurements made in the present study by a function of two separate terms in the form

$$(Nu)_f = g\,[a\,(Re)_f^{\alpha} + b\,(Re)_f^{\beta}] \qquad (10)$$

is adopted, the resulting equation must satisfy the following three conditions:

I. It must correlate the experimental heat-transfer data. Since McAdams' correlation accurately represents the experimental results, this condition is equivalent to stating that the new correlation must coincide *numerically* with McAdams' correlation.
II. The values of α and β should agree with theory so far as the theory is known.
III. The ratios of $a(Re)_f^{\alpha}$ to $b(Re)_f^{\beta}$ must agree with reported measured ratios of the contributions to heat transfer from the front and back portions of a cylinder in crossflow.

Condition I suggests that the new correlation be cast in a form similar to McAdams' correlation, equation (3), with the Reynolds number term replaced by two terms as follows:

$$(Nu)_f = [0{\cdot}35 + a\,(Re)_f^{\alpha} + b\,(Re)_f^{\beta}]\ (Pr)_f^{0{\cdot}30}. \qquad (11)$$

The insertion of the constant 0·35 and the quantity $(Pr)_f^{0\cdot30}$ in equation (11) implies that the temperature-dependence of the fluid properties is correctly accounted for by McAdams' correlation. This assumption is made because the temperature differences achieved in the present investigation are relatively small, and therefore these experiments do not provide sufficient information to permit an independent determination of the functional dependence of $(Nu)_f$ on $(Pr)_f$.

The theory of convective heat transfer to laminar flows is quite well known and predicts that the value of α is $\frac{1}{2}$. However, no theoretical solution for heat transfer in the region of separated flow on the rear portion of a cylinder in crossflow has been achieved to date, and hence the value of β in equation (11) cannot as yet be determined from purely theoretical considerations. Douglas and Churchill have recommended that β be taken equal to 1; Richardson suggests that $\beta = 0{\cdot}67$, which, as will be shown, is closer to the value determined here. If the known value for α is inserted into equation (11), one obtains:

$$(Nu)_f = [0{\cdot}35 + a\,(Re)_f^{0{\cdot}50} + b\,(Re)_f^{\beta}]\,(Pr)_f^{0{\cdot}30}. \qquad (12)$$

The problem now is to determined, if possible,

Table 2. Comparison of experimentally determined Nusselt numbers with predicted values calculated from three different correlation equations

	A	*B*	Deviation (%)	*C*	*D*	Deviation (%)	*E*	*F*	Deviation (%)
Run No.	$(Nu)_f$ Experimental	$(Nu)_f$ Calculated from Eq. (3)	$\frac{B - A}{A}(100)$	$(Nu)_f$ Experimental	$(Nu)_f$ Calculated from Eq. (14)	$\frac{D - C}{C}(100)$	$(Nu)_a$ Experimental	$(Nu)_a$ Calculated from Eq. (7′)	$\frac{F - E}{E}(100)$
1	260·5	261·0	+0·2	260·5	261·6	+0·4	262·0	452·0	+72·5
2	148·3	146·5	−1·2	148·3	143·6	−3·2	149·0	229·5	+54·0
3	116·1	119·7	+3·1	116·1	116·7	+0·6	116·9	181·6	+55·3
4	235·0	241·0	+2·6	235·0	231·6	−1·4	236·0	416·0	+76·3
5	198·5	198·7	+0·1	198·5	198·9	+0·2	199·5	330·0	+65·4
6	296·0	293·5	−0·8	296·0	297·1	+0·4	297·5	517·0	+73·7
	Mean deviation = 1·3%			Mean deviation = 0·9%			Mean deviation = +66·2%		

numerical values for the constants a, b and β such that these values will correlate the experimental data obtained in the present study and *simultaneously* satisfy Condition III.

McAdams presents in his textbook the results of measurements made by four different investigators of the local Nusselt number around a cylinder in crossflow in air at a Reynolds number of 39 600. These data indicate that for $(Re)_f = 39\,600$ the contributions to the total heat transfer from the front and rear portions of a cylinder in crossflow in air are very nearly equal. Translated into the language of equation (12) and Condition III, these local heat-transfer data imply that

$$a\,(39\,600)^{0\cdot50} = b\,(39\,600)^{\beta}. \qquad (13)$$

Thus, the problem has been reduced to determining values of a, b and β in equation (12), which will correlate the experimental Nusselt numbers and simultaneously satisfy equation (13). The values of a, b and β were determined to be 0·34, 0·15 and 0·58 respectively. The resulting correlation equation is:

$$(Nu)_f = [0\cdot35 + 0\cdot34\,(Re)_f^{0\cdot50} + 0\cdot15\,(Re)_f^{0\cdot58}]\,(Pr)_f^{0\cdot30}. \qquad (14)$$

The deviations between the values of $(Nu)_f$ predicted by equation (14) and the experimental results are given in Table 2; the maximum deviation for any single data point is 3·2 per cent, and the mean deviation is 0·9 per cent. Equation (14) is plotted in Fig. 5, where it is shown to agree closely with the McAdams equation.

In order to check the validity of the experimentally determined constants, particularly the value $\beta = 0\cdot58$, some additional published local Nusselt number data were integrated, and the ratios of the contributions to the total Nusselt number from the leading and trailing portions of a cylinder were calculated; these values are listed in Table 3, together with comparable calculated ratios of $[0\cdot34\,(Re)_f^{0\cdot50}/0\cdot15\,(Re)_f^{0\cdot58}]$. The agreement between experimental and calculated ratios is good for Reynolds numbers between 2×10^4 and 10^5. This agreement not only confirms the correctness of the values determined for a, b and β for

$$2 \times 10^4 < (Re)_f < 10^5,$$

but also indicates that the constants a, b and β have been uniquely determined in this range of $(Re)_f$. For values much above 10^5 or far below 2×10^4, $[0\cdot34\,(Re)_f^{0\cdot50}/0\cdot15\,(Re)_f^{0\cdot58}]$ is not precisely equal to corresponding experimentally determined ratios; thus, for $(Re)_f = 1\cdot7 \times 10^5$, $[0\cdot34\,(Re)_f^{0\cdot50}/0\cdot15\,(Re)_f^{0\cdot58}] = 0\cdot86$, but the comparable value obtained by integrating Schmidt and Wenner's [14] local heat-transfer data is 0·72.

The use of local heat-transfer data taken in *air*, such as Schmidt and Wenner's, to infer ratios such as $[a\,(Re)_f^{\alpha}/b\,(Re)_f^{\beta}]$ in *water* violates the law of dynamic similarity stated in the section Survey of the Literature. This violation has been

Table 3. Comparison of $\{[0\cdot34\,(Re)_f{}^{0\cdot50}]/[0\cdot15\,(Re)_f{}^{0\cdot58}]\}$ *with corresponding experimental ratios*

Reynolds number	Ratio of experimentally determined heat transfer through laminar boundary layer on front portion of cylinder to heat transfer through separated region on back portion of cylinder	Source of experimental data	$\left[\frac{0\cdot34(Re)_f{}^{0\cdot50}}{0\cdot15(Re)_f{}^{0\cdot58}}\right]$ [See Eq. (14)]
20 000	1·07	By Lohrisch in reference 13	1·03
39 600	1·00	By Small, Lohrisch, Klein, Drew and Ryan in reference 5	1·00
101 000	0·80	By Schmidt and Wenner in reference 5	0·85

committed here because no comparable low-turbulence local heat-transfer data for water are available. The error incurred by this procedure is probably small, because the *ratio* of the heat transfer from the front and back of a cylinder in air at a given Reynolds number is probably very nearly the same as the *ratio* for water, even if the magnitudes of the contributions differ somewhat for air and water.

DISCUSSION OF RESULTS

The experimental results of the present work are in very close agreement with McAdams' correlation, equation (3). The closeness of this agreement may, to some extent, be fortuitous, because the correction applied to the thermocouple readings, Δt_r, is approximate and to some extent arbitrary. It is quite conceivable that this temperature correction is sufficiently in error to alter the experimentally determined values of $(Nu)_f$ by as much as 5 per cent. Also, there is some question as to which is the optimum method for correcting for blockage; thus, if the blockage correction were computed on the basis of "solid blocking" and "wake blocking," instead of on the basis of the "mean area," the calculated values of $(Nu)_f$ would be about 3 per cent less than reported here. However, agreement within 5 per cent—or even 10 per cent—is generally considered to be acceptable in heat-transfer work, and hence it is concluded that the experimental results presented herein indicate that the McAdams' correlation is valid in the range of Reynolds numbers from 10^4 to 10^5.

The correlation of McAdams, the conclusions of Purves and Brodkey, and the experimental data of the present work are all in basic agreement. The results of these three studies differ significantly from the correlation suggested by Perkins and Leppert, equation (7′), which gives values that are too high. Perkins and Leppert themselves point out that their Nusselt number data are high compared to those of Davis, [1] and they suggest that the reason for this might be the existence of higher turbulence levels in their experiments than in Davis'.

The influence of turbulence on heat transfer is known to be considerable [15–20] but it is not yet sufficiently understood to permit the quantitative calculation of its magnitude. In Perkins and Leppert's experiments, screens were placed upstream of the test cylinder for the purpose of breaking up the flow and producing as flat a velocity profile as possible. No effort was made to measure the turbulence level in the present work, but it was doubtlessly considerably lower than in Perkins and Leppert's setup, since no screens were used here and the flow passed through calming sections before entering the duct wherein the test cylinder was located. Thus, a large part of the discrepancy between the results of Perkins and Leppert and the present work may be attributed to turbulence effects. And yet, although turbulence effects are important, a study of the literature on this subject suggests that turbulence effects might account for a deviation of 20–30 per cent between the results of Perkins and Leppert and the present work, but not as much as 54–76 per cent, as is actually the case.

Part of the discrepancy between the results of Perkins and Leppert and the present study may be due to complex secondary flows and boundary layer effects near the walls of the rectangular water tunnel in which their experiments were conducted. Such effects were avoided in the present study by using a circular water tunnel and the guarded test section technique.

CONCLUSIONS

This paper presents the results of an experimental investigation of heat transfer by forced convection from a cylinder to water in crossflow in the range $10^4 < (Re)_f < 10^5$. The diameter of the test cylinder was 0·4375 in, and the temperature difference varied from 4 to 10 degF. The experimental data are in close agreement with the McAdams correlation,

$$(Nu)_f = [0{\cdot}35 + 0{\cdot}56\,(Re)_f^{0{\cdot}52}]\,(Pr)_f^{0{\cdot}30}. \qquad (3)$$

McAdams derived equation (3) from heat-transfer data in the range

$$10^{-1} < (Re)_f < 2 \times 10^2.$$

Purves and Brodkey infer from the Colburn analogy that equation (3) is valid in the range $10^2 < (Re)_f < 10^4$; the present study shows experimentally that McAdams' correlation is also valid in the range $10^4 < (Re)_f < 10^5$. It is concluded, therefore, that McAdams' relation holds over the entire range of Reynolds numbers from 10^{-1} to 10^5.

The correlation of Perkins and Leppert,

$$(Nu)_a \left(\frac{\mu_s}{\mu_a}\right)^{0\cdot25} = [0\cdot30\,(Re)_a^{0\cdot50} + 0\cdot10\,(Re)_a^{0\cdot67}]\,(Pr)_a^{0\cdot40}, \quad (7)$$

predicts Nusselt numbers which are, on the average, more than 60 per cent higher than the corresponding experimental values obtained in the present study. It is suggested that free stream turbulence, secondary flows and boundary-layer effects associated with the rectangular duct in the apparatus of Perkins and Leppert are the cause of this discrepancy; these effects were eliminated from the present study by using a circular water duct with calming sections and the guarded test section technique for measuring convective heat transfer.

A second correlation is presented herein, namely,

$$(Nu)_f = [0\cdot35 + 0\cdot34\,(Re)_f^{0\cdot50} + 0\cdot15\,(Re)_f^{0\cdot58}]\,(Pr)_f^{0\cdot30}, \quad (14)$$

which agrees closely with McAdams' correlation. However, equation (14) has the added virtue of representing the separate contributions to $(Nu)_f$ by the leading and trailing portions of the cylinder. Equations (3) and (14) predict $(Nu)_f$ with equal accuracy; but equation (14) is preferable because it represents more realistically the actual physical heat-transfer process. The representation referred to in the preceding statement is precise for the range of Reynolds number between 2×10^4 and 10^5; it does not, nor can it be expected to, hold for much higher Reynolds numbers (near critical or supercritical), or for much lower Reynolds numbers, for which a vortex street is not established.

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Résumé—Cet article présente les résultats d'une recherche expérimentale sur le transport de chaleur par convection forcée à partir d'un cylindre dans un écoulement transversal d'eau dans le gamme des nombres de Reynolds allant approximativement de 10^4 à 10^5. Le diamètre du cylindre essayé était de 1,11 cm et la différence de température variait de 2,22 à 5,55 degC. Les données expérimentales sont en bon accord (déviation moyenne = 1 %) avec la corrélation de McAdams, c'est-à-dire,

$$(Nu)_f = [0{,}35 + 0{,}56\,(Re)_f^{0{,}52}]\,(Pr)_f^{0{,}30}.$$

McAdams a obtenu son équation de corrélation à partir des données expérimentales prises dans une gamme limitée de nombres de Reynolds de l'écoulement transversal allant de 10^{-1} à 2×10^2. Purves et Brodkey ont déduit à partir de l'analogie de Colburn que la corrélation de McAdams est valable dans la gamme des nombres de Reynolds de 10^2 à 10^4. Donc, sur la base des données expérimentales actuelles, on conclut que l'équation de McAdams est valable dans une gamme étendue de nombres de Reynolds allant de 10^{-1} à 10^5.

Les données expérimentales actuelles sont bien représentées également par

$$(Nu)_f = [0{,}35 + 0{,}34\,(Re)_f^{0{,}50} + 0{,}15\,(Re)_f^{0{,}58}]\,(Pr)_f^{0{,}30}.$$

Dans la dernière équation, le terme en $(Re)_f^{0{,}50}$ représente le transport de chaleur à travers la couche limite laminaire sur la portion avant du cylindre, et le terme en $(Re)_f^{0{,}58}$ provient de la contribution au transport de chaleur total de la partie arrière de cylindre, où le décollement se produit. Chacune des deux équations données ici peut être utilisée pour des calculs de projet, mais la seconde équation est préférable parce qu'elle est reliée plus logiquement aux processus physiques en cause.

Zusammenfassung—Diese Arbeit bringt die Ergebnisse einer experimentellen Untersuchung des Wärmeüberganges durch Zwangskonvektion von einem Zylinder an Wasser im Kreuzstrom in einem Bereich der Reynoldszahlen von ungefähr 10^4 bis 10^5. Der Durchmesser des Versuchszylinders betrug 1,11 cm und die Temperaturdifferenz änderte sich von 2,22 bis 5,55 degC. Die Versuchsdaten stimmen sehr gut (mittlere Abweichung = 1 %) mit der Gleichung von McAdams überein, die lautet

$$(Nu)_f = [0{,}35 + 0{,}56\,(Re)_f^{0{,}52}]\,(Pr)_f^{0{,}30}.$$

McAdams leitete seine Beziehungsgleichung aus Versuchsdaten ab, die in einem begrenzten Bereich von Reynoldszahlen bei Kreuzstrom von 10^{-1} bis 2×10^2 bestimmt wurden. Purves und Brodkey folgern aus der Analogie Colburns, dass McAdams Beziehung für den Bereich der Reynoldszahlen von 10^2 bis 10^4 gilt. Deshalb wird mit den vorliegenden Versuchsdaten als Grundlage geschlossen, dass die Gleichung von McAdams in dem erweiterten Bereich der Reynoldszahlen von 10^{-1} bis 10^5 gültig ist.

Die vorliegenden Versuchsdaten werden ebenfalls gut dargestellt durch

$$(Nu)_f = [0{,}35 + 0{,}34\,(Re)_f^{0{,}50} + 0{,}15\,(Re)_f^{0{,}58}]\,(Pr)_f^{0{,}30}.$$

In der letztgennanten Gleichung stellt der Ausdruck $(Re)_f^{0{,}50}$ den Wärmeübergang durch die laminare Grenzschicht an der Vorderseite des Zylinders dar und der Ausdruck $(Re)_f^{0{,}58}$ erklärt den Beitrag der Zylinderrückseite zum gesamten Wärmeübergang, wo Ablösung erfolgt. Jede der beiden hier aufgeführten Gleichungen kann für Auslegungsberechnungen verwendet werden, aber die zweite Gleichung ist in dem Sinne vorzuziehen, dass sie mit den massgebenden physikalischen Vorgängen logischer verbunden ist.

Аннотация—В данной статье представлены результаты экспериментального исследования переноса тепла вынужденной конвекцией от цилиндра к воде в поперeченом потоке в диапазоне чисел Рейнольдса от 10^4 до 10^5. Диаметр исследуемого цилиндра составлял 0,4375 дюйма, разность температур изменалась от 4 до 10 °*F*. Экспериментальные данные находятся в хорошем согласии (среднее отклонение = 1 %) с аппроксимацией, предложенной Мак-Адамсом, а именно

$$(Nu)_f = [0{,}35 + 0{,}56\,(Re)_f^{0{,}52}]\,Pr_f^{0{,}30}$$

Мак-Адамс получил свое уравнение, аппроксимирующее экспериментальные данные, полученные при ограниченном диапазоне чисел Рейнольдса для поперечного потока от 10^{-1} до 2×10^2 Первз и Бродки из аналогии Колберна сделали вывод, что формула Мак-Адамса справедлива в диапазоне чисел Рейнольдса от 10^2 до 10^4. Поэтому на основе настоящих экспериментальных данных делается вывод, что уравнение Мак-Адамса справедливо в большем диапазоне чисел Рейнольдса: от 10^{-1} до 10^5.

Настоящие экспериментальные данные также хорошо описываются уравнением

$$(Nu)_f = [0{,}35 + 0{,}34\,(Re_f^{0{,}50} + 0{,}15\,(Re)_f^{0{,}58}]\,(Pr)_f^{0{,}30}.$$

В последнем уравнении член $(Re)_f^{0{,}50}$ представляет собой перенос тепла через ламинарный пограничный слой на лобовой части цилиндра, а член $(Re)_f^{0{,}58}$ означает долю, вносимую в общий перенос тепла задней частью цилиндра, где происходит отрыв потока. Любое из двух приведенных здесь уравнений может быть использовано для проектных расчетов, но второе уравнение следует предпочесть в том смысле, что оно более логично связано с происходящими физическими процессами.